Midterm exam Partial Differential Equations

Block 2B, 2022–2023

university of groningen

INSTRUCTIONS TO CANDIDATES

1. Attempt all 4 questions in this test. The total number of points available is 100. You will get 10 points for free.
2. The number of points you can get for each question is shown next to it.
3. In answering the questions in this paper it is particularly important to show your argumentation. The total number of points will only be given for full and detailed answers.
4. Simple pocket calculators are allowed at this exam. Other electronic devices such as graphical/programmable calculators, tablets, laptops and mobile phones are not.
5. Books, notes and formula sheets are all not allowed.
6. Please make sure that all pages you hand in have your name and student ID on them.

DO NOT REMOVE THIS DOCUMENT FROM THE EXAMINATION ROOM.

1 **[20 points]** Use a suitable change of variables to find the general solution of $u_{xx} + 3u_{xy} - 4u_{yy} = 4y + 5x$.

2 Consider the initial value problem $u_t + x^2 u_x = x^2$, $u(0, x) = e^x$.

a. **[10 points]** Rewrite the initial value problem as $v_t + x^2 v_x = 0$ for a suitable initial condition $v(0, x) = f(x)$, where $v(t, x) = u(t, x) + g(x)$ and f and g are to be determined scalar functions.

b. **[10 points]** Solve the initial value problem $u_t + x^2 u_x = x^2$, $u(0, x) = e^x$.

3 Consider the initial value problem $u_t = 5u_{xx}$, $u(t, 0) = u(t, 17) = 0$, $u(0, x) = 2023x(17 - x)$, $t \geq 0$.

a. **[10 points]** Let u_1, u_2 solve the initial value problem. Write out an initial value problem for $v := u_1 - u_2$ and use this to prove that $u_1 = u_2$.

b. **[10 points]** Prove that $E(t) = \int_0^{17} u(t, x)^2 dx$ is a decreasing function.

4 **[30 points]** Let $c > 0$ be a constant. Solve the initial value problem

$$u_{tt} = c^2 u_{xx}, \quad u_t(0, x) = x, \quad u(0, x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ 1 & \text{if} \quad 0 \leq x < 2 \\ 2x & \text{if} \quad x \geq 2 \end{cases},$$

where $t \geq 0$. **Hint:** Divide the $(t, x)-$plane into different regions and give your solution per region.